Lecture 4

Tuesday, September 6, 2016 8:52 AM

- <u>Ex</u>

$\cos(\underbrace{\arcsin 0}) = ?$ $(\arcsin = \sin^{-1})$

$\arcsin 0 = x \overset{\text{DEF}}{\Leftrightarrow} \sin x = 0$

$\Leftrightarrow x = 0$

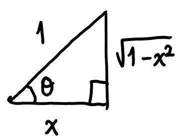
- $\cos 0 = 1$

$\cos(\arcsin 0) = 1$

<u>Ex</u> Simplify the expression $\tan(\cos^{-1} x)$

Set $\cos^{-1} x = \theta \Leftrightarrow \cos\theta = x$

$\tan(\theta) = \dfrac{\sqrt{1-x^2}}{x}$



$\tan(\cos^{-1} x) = \dfrac{\sqrt{1-x^2}}{x}$ ☒

Review of Limits

<u>DEF</u> Suppose $f(x)$ is a func defined near a (except possibly at $x = a$)

$\lim\limits_{x \to a} f(x) = L$ if we can make $f(x)$ arbitrarily close to L by taking x sufficiently close to a.

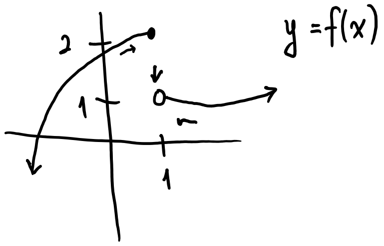
One Sided Limits

$\lim\limits_{x \to a^-} f(x) = L$

Def is almost the same as above except we require that $x < a$.

$\lim_{x \to a^+} f(x) = L$ $(x > a)$

Ex



$\lim_{x \to 1^-} f(x) = 2$ $\lim_{x \to 1^+} f(x) = 1$

$\lim_{x \to 1} f(x)$ DNE

DEF $\lim_{x \to a} f(x) = L$ if and only if

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$

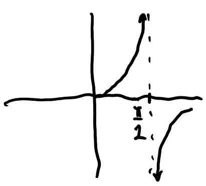
Infinite Limits

Let f be defined near a

Then $\lim_{x \to a} f(x) = -\infty$

means that $f(x)$ can be made arbitrarily ~~large~~ small by taking x sufficiently close to a.

Ex $\lim_{x \to \frac{\pi}{2}} \tan x$ DNE



$\lim_{x \to \frac{\pi}{2}^-} f(x) = \infty$

$\lim_{x \to \frac{\pi}{2}^+} f(x) = -\infty$

$\tan x$ has a vertical asymptote

at $x = \frac{\pi}{2}$.

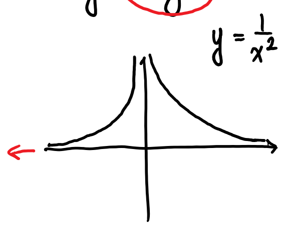
Limits at Infinity

Let f be a funct defined on (a, ∞) $(-\infty, a)$

Then $\lim_{x \to \infty} f(x) = L$ means that

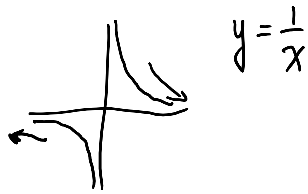
$f(x)$ can be made arbitrarily close to L by requiring that x be sufficiently large. small

Ex



$$\lim_{x \to \infty} \frac{1}{x^2} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x^2} = 0$$



Thm Let $r > 0$ be a rational number, then

$$\lim_{x \to \infty} \frac{1}{x^r} = 0$$

If $r > 0$ is a rat'l number and x^r is defined for all x, then

- $$\lim_{x \to -\infty} \frac{1}{x^r} = 0$$

$$\boxed{x^{1/2} = \sqrt{x}}$$

Horizontal Asymptote (H.A)

A line $y = L$ is called a H.A. to the curve $y = f(x)$ if

$$\lim_{x \to \infty} f(x) = L \quad \text{or} \quad \lim_{x \to -\infty} f(x) = L$$

- Limit Laws (Pg 95, 2.3)

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Ex $\lim_{h\to 0} \frac{(2+h)^2-4}{h}$

Plug in $h=0$

$$\frac{(2+0)^2-4}{0} = \frac{4-4}{0} = \frac{0}{0} \quad \text{(Algebra)}$$

- $$\lim_{h\to 0} \frac{(2+h)^2-4}{h} = \lim_{h\to 0} \frac{\cancel{4}+4h+h^2\cancel{-4}}{h}$$

$$= \lim_{h\to 0} \frac{4h+h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 4+h = 4$$

Ex $\lim_{x\to\infty} \frac{2x^3-x^2}{x-x^3}$

Divide the numerator and denom by the largest power in the den.

$$\lim_{x\to\infty} \frac{\frac{2x^3}{x^3} - \frac{x^2}{x^3{}_1}}{\frac{x^1}{x^3{}_2} - \frac{x^3}{x^3}} = \lim_{x\to\infty} \frac{2-\frac{1}{x}}{\frac{1}{x^2}-1}$$

Limit Laws DIY

$$= \frac{\lim_{x\to\infty} 2 - \lim_{x\to\infty} \frac{1}{x^1}}{\lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} 1} = \frac{2-0}{0-1} = \frac{2}{-1} = -2$$

$\lim_{x\to-\infty} \frac{1}{x^r} = 0$ if x^r is defined for all x.

$\lim_{x\to-\infty} \frac{1}{x^{1/2}}$ ×

$\lim_{x\to\infty} \frac{1}{x^r}$

$x\to\infty$.

Continuity (2.5)

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A function $f(x)$ is continuous at a number a if $\lim_{x \to a} f(x) = f(a)$.

1) $f(a)$ is defined. ✓

2) $\lim_{x \to a} f(x)$ exists. ✓

3) $\lim_{x \to a} f(x) = f(a)$

$x \to \infty$. $x \to -\infty$ x^2 $\lim_{x \to \infty} \frac{1}{x^r}$ x

$\frac{1}{\sqrt{x}}$ $(-2000)^{1/2}$

f^{-1}

$y = \frac{5e^x}{e^x - 1}$

$ye^x = 5e^x + y$

$ye^x - 5e^x = y$

$e^x(y-5) = y$

$e^x = \frac{y}{y-5}$

$x = \ln\left(\frac{y}{y-5}\right)$

$f^{-1}(x) = \ln\left(\frac{x}{x-5}\right)$

$x \text{ in } [5, 7)$ $[a, \infty)$

$5 \leq x < 7$ $a \leq x$

$\lim_{x \to \pm\infty} \frac{1}{x^r} = 0$ $x^{-1/2} = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$

$x^{-r} = \frac{1}{x^r}$